A deep learning approach to universal cycles

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Examples of universal cycles. Given a finite alphabet and a window size m, a *de Bruijn sequence* is a word containing as a factor each possible word of length m exactly once. An example of de Bruijn sequence on the alphabet $\{0, 1\}$ for $m = 3$ is

$$0001011100.$$

Constructing de Bruijn sequences is a classic problem with many interesting solutions [deb (2024)]. The notion has been generalized to *universal cycles* by [Chung et al. (1992)]. Without going into the formal definition, let us give an example.

Consider a robot moving on the colored grid from Figure 1. At any position, the robot perceives only a 3 by 3 square window around it. The measurement device of the robot is simple and retrieves only the multiset of colors in the window, instead of the exact pattern. The grid has been colored to ensure that no two windows contain the same multiset of colors. Thus, the robot can always deduce its position from the multiset it measures. We refer to the mapping as a *color mapping* and the code as a *color multiset code* [Chen et al. (2024b)]. Some of the authors of the present internship proposal have worked on this problem and designed an asymptotically optimal solution [Chen et al. (2024a)]. In this problem, the goal is to color a large grid without two window corresponding to the same measurement value. This is called *packing*. The dual problem where we color a small grid while ensuring that each possible measurement value is reached on at least one window is called *covering*.

Deep learning approach. The goal of the internship is to develop a deep learning algorithm for universal cycles packing and covering, solving problems such as the ones described in the previous paragraph. One of the motivations is the observation that practical applications rarely fit the mathematical modeling exactly. For example, the grid and window might have irregular or circular shapes. While designing a scheme by

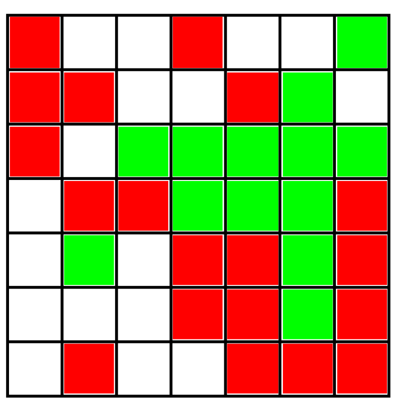

Figure 1: A colored grid where no two 3 by 3 squares contain the same multiset of colors.

hand to cover every possible variant is tedious, an algorithm based on deep learning is flexible, requiring only to adapt the reward function. The drawback of the deep learning approach is that the solution comes without any theoretical guarantee on its quality. There are many different instances of universal cycles in the scientific literature. Some are well studied (such as de Bruijn sequences), others present open problems. The performance of our deep learning algorithm where optimal bounds are known will hint to the quality of the output on problems where theoretical bounds have yet to be proven. This internship is also an opportunity to investigate how to apply neural networks to symbolic problems.

Related work. Deep learning has been applied to several combinatorial optimization problems [Sun and Yang (2023)], such as the design of fast matrix multiplication algorithms [Fawzi et al. (2022)], graph coloring [Huang et al. (2019)]. We intend to take advantage of the lessons learned in those works.

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