Quantum states are relative to observers!

… and this is important for quantum networks

LINCS reading group “Network Theory”

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Quantum states are relative to observers!

Pseudo-paradoxes in quantum physics

Some pseudo-paradoxes in quantum physics

- Schrödinger's cat: dead and alive at the same time?
- Einstein-Podolsky-Rosen (EPR) dilemma: spooky action at distance?
  - Measuring an entangled qubit in one location instantaneously changes its state of the other qubit in another location

Misunderstanding of what is a quantum state leads to wrong statements

- Special relativity is wrong because of the EPR "paradox"?
  - Special relativity (and general relativity) has always been true since 1905 (resp., since 1916)…
- Transmission of "information" faster than light with entanglement?
  - In contradiction with the no-go theorem about "no-faster-than-light signaling" in quantum physics…
  - Quantum physics has always been true since years 1920s…
- The so-called "measurement problem" in quantum physics?
  - Still an open question in quantum physics, but a better understanding of what is a quantum state should help…

What is a quantum state?

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Some mathematical notations

Dirac notation ("bra-ket" notation)

- See https://fr.wikipedia.org/wiki/Notation_bra-ket
- Quantum state ψ = line (ray) in Hilbert space $V = \mathbb{C}^d$ (N qubits: $d = 2^N$)
- Vector representing quantum state ψ: $|\psi\rangle \in V$ ("ket")
- Corresponding "bra": $\langle\psi| \in dual\ (V)$
  - Rule: $\lambda|\varphi\rangle + \mu|\psi\rangle \leftrightarrow \lambda^\star\langle\varphi| + \mu^\star\langle\psi|$
- Hermitian product (~ scalar product in Euclidian space): $\langle\varphi|\psi\rangle = \langle\varphi| \cdot |\psi\rangle = \langle\psi|\varphi\rangle^\star$

$$\langle\, (\lambda|\varphi\rangle + \mu|\psi\rangle) \mid (\lambda'|\varphi'\rangle + \mu'|\psi'\rangle\,)\rangle = (\lambda^\star\,\langle\varphi| + \mu^\star\,\langle\psi|) \cdot (\lambda'|\varphi'\rangle + \mu'|\psi'\rangle\,)$$
$$= \lambda^\star\lambda'\langle\varphi|\varphi'\rangle + \lambda^\star\mu'\langle\varphi|\psi'\rangle + \mu^\star\lambda'\langle\psi|\varphi'\rangle + \mu^\star\mu'\langle\psi|\psi'\rangle$$

- Linear operators: $\sum c_k|\psi_k\rangle\langle\varphi_k|$, with $|\psi_k\rangle\langle\varphi_k||\chi\rangle = |\psi_k\rangle\langle\varphi_k|\chi\rangle = \langle\varphi_k|\chi\rangle \cdot |\psi_k\rangle$
- Orthogonal projector on a state ψ : $|\psi\rangle\langle\psi|$ with $\langle\psi|\psi\rangle = 1$
  - $|\psi\rangle\langle\psi||\psi\rangle = \langle\psi|\psi\rangle \cdot |\psi\rangle = |\psi\rangle$
  - $\forall|\varphi\rangle \in V$ such as $\langle\psi|\varphi\rangle = 0$, i.e., $|\varphi\rangle \perp |\psi\rangle$, $|\psi\rangle\langle\psi||\varphi\rangle = \langle\psi|\varphi\rangle \cdot |\psi\rangle = 0 \cdot |\psi\rangle = 0$

Quantum states are relative to observers!

Qubits: bases, Bloch sphere

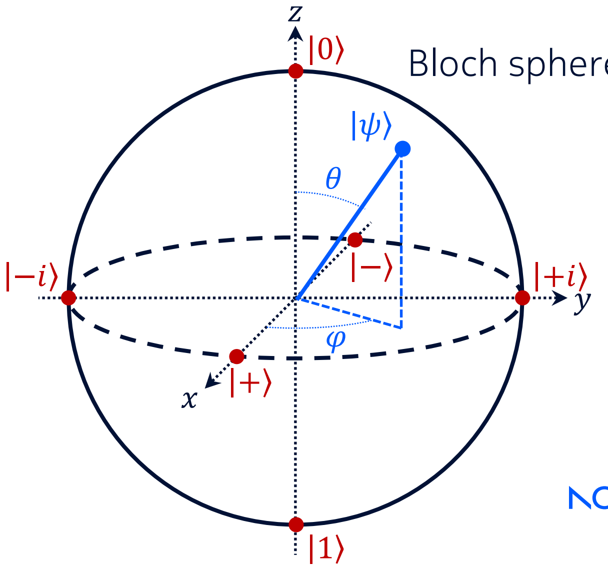
States in quantum physics (mathematical modeling)

- A quantum state ψ can be represented by a vector in a complex Hilbert space: $|\psi\rangle \in V$ ("ket"), up to normalization factor $\langle\varphi|\varphi\rangle = 1$ and a global phase factor, i.e., $|\varphi\rangle \cong e^{i\alpha}|\varphi\rangle$
- ψ = line (ray) in the Hilbert space: the set of quantum states is the projective space $\mathrm{P}(V)$

Qubits = lines in a Hilbert space V with $\dim(V) = 2$

- Quantum state ψ : $|\psi\rangle = \left(e^{i\alpha}\right) \cdot \left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$
  with $\theta \in [0,\pi]$ and $\varphi \in [0,2\pi[$
- Some bases (orthonormal bases):
  - Computational basis $(|0\rangle, |1\rangle)$ on the z axis
  - Basis $(|+\rangle, |-\rangle)$ on the x axis with $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
  - Basis $(|+i\rangle, |-i\rangle)$ on the y axis with $|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$ and $|-i\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

Bloch sphere = $\mathrm{P}(\mathbb{C}^2)$



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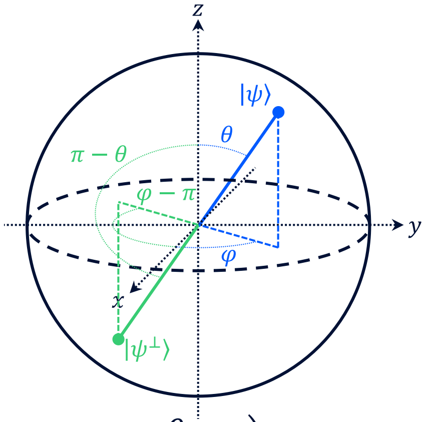
Qubits: orthogonal state



Qubit state ψ represented by the vector $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$

Its "orthogonal state" $\psi^{\perp}$ can be represented by the vector:

$$|\psi^{\perp}\rangle = e^{i\beta}\cdot\left(-e^{-i\varphi}\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle\right) = e^{i(\beta-\varphi+\pi)}\cdot\left(\cos\frac{\pi-\theta}{2}|0\rangle + e^{i(\varphi-\pi)}\sin\frac{\pi-\theta}{2}|1\rangle\right)$$

such as $(|\psi\rangle, |\psi^{\perp}\rangle)$ is another basis of V:

- Computational basis : $\langle 0|0\rangle = \langle 1|1\rangle = 1$ and $\langle 0|1\rangle = \langle 1|0\rangle = 0$
- $\langle\psi|\psi\rangle = \left\langle\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\middle|\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right\rangle = \cos^2\frac{\theta}{2} + e^{-i\varphi}e^{i\varphi}\sin^2\frac{\theta}{2} = 1$
- $\langle\psi^{\perp}|\psi^{\perp}\rangle = e^{-i\beta}e^{i\beta}\left\langle-e^{-i\varphi}\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle\middle|-e^{-i\varphi}\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle\right\rangle = e^{+i\varphi}e^{-i\varphi}\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} = 1$
- $\langle\psi^{\perp}|\psi\rangle = \left\langle-e^{-i\varphi}\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle\middle|\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right\rangle = -e^{+i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\frac{\theta}{2}e^{i\varphi}\sin\frac{\theta}{2} = 0$

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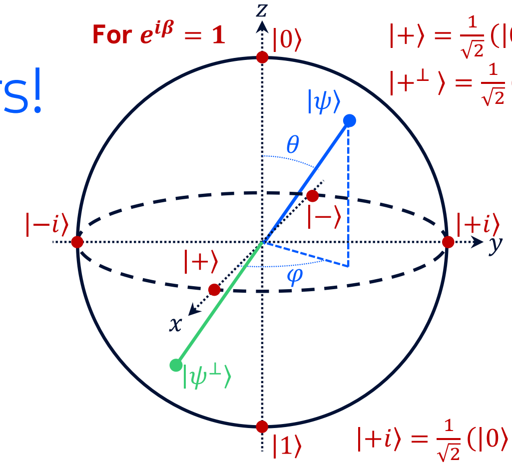
Qubits: orthogonal state – Basis transformations

For $e^{i\beta} = 1$



$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|+^{\perp}\rangle = \frac{1}{\sqrt{2}}(-|0\rangle + |1\rangle) = -|-\rangle$

$|0^{\perp}\rangle = |1\rangle$

$|\psi^{\perp\perp}\rangle = -|\psi\rangle$

$|+i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

$|+i^{\perp}\rangle = \frac{1}{\sqrt{2}}(i|0\rangle + |1\rangle) = -i|-i\rangle$

Computational basis $(|\psi\rangle, |\psi^{\perp}\rangle)$ in the basis $(|0\rangle, |1\rangle)$:

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \qquad \textbf{(1)}$$

$$|\psi^{\perp}\rangle = e^{i\beta}\cdot\left(-e^{-i\varphi}\sin\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}|1\rangle\right) \qquad \textbf{(2)}$$

Computational basis $(|0\rangle, |1\rangle)$ in the basis $(|\psi\rangle, |\psi^{\perp}\rangle)$:

$$\left(\cos\frac{\theta}{2}\right)\times\textbf{(1)} \Rightarrow \cos\frac{\theta}{2}|\psi\rangle = \cos^2\frac{\theta}{2}|0\rangle + \cos\frac{\theta}{2}e^{i\varphi}\sin\frac{\theta}{2}|1\rangle \qquad \textbf{(3)}$$

$$\left(-e^{-i\beta}e^{i\varphi}\sin\frac{\theta}{2}\right)\times\textbf{(2)} \Rightarrow -e^{-i\beta}e^{i\varphi}\sin\frac{\theta}{2}|\psi^{\perp}\rangle = \sin^2\frac{\theta}{2}|0\rangle - e^{i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2}|1\rangle \qquad \textbf{(4)}$$

$$\textbf{(3)} + \textbf{(4)} \Rightarrow |0\rangle = \cos\frac{\theta}{2}|\psi\rangle - e^{-i\beta}e^{i\varphi}\sin\frac{\theta}{2}|\psi^{\perp}\rangle \qquad \textbf{(1')}$$

$$\left(e^{-i\varphi}\sin\frac{\theta}{2}\right)\times\textbf{(1)} \Rightarrow e^{-i\varphi}\sin\frac{\theta}{2}|\psi\rangle = e^{-i\varphi}\sin\frac{\theta}{2}\cos\frac{\theta}{2}|0\rangle + \sin^2\frac{\theta}{2}|1\rangle \qquad \textbf{(5)}$$

$$\left(e^{-i\beta}\cos\frac{\theta}{2}\right)\times\textbf{(2)} \Rightarrow e^{-i\beta}\cos\frac{\theta}{2}|\psi^{\perp}\rangle = -\cos\frac{\theta}{2}e^{-i\varphi}\sin\frac{\theta}{2}|0\rangle + \cos^2\frac{\theta}{2}|1\rangle \qquad \textbf{(6)}$$

$$\textbf{(5)} + \textbf{(6)} \Rightarrow |1\rangle = e^{-i\varphi}\sin\frac{\theta}{2}|\psi\rangle + e^{-i\beta}\cos\frac{\theta}{2}|\psi^{\perp}\rangle \qquad \textbf{(2')}$$

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Qubits: observables, measurement

Observables on qubits:

- $\forall c \in V, M_{\psi,\lambda_\psi,\lambda_{\psi^\perp}} = \lambda_\psi |\psi\rangle\langle\psi| + \lambda_{\psi^\perp} |\psi^\perp\rangle\langle\psi^\perp|$ with $\lambda_\psi \in \mathbb{R}$ and $\lambda_{\psi^\perp} \in \mathbb{R} \setminus \{\lambda_\psi\}$
- Spin observable: $S_\psi = \frac{\hbar}{2}\left(|\psi\rangle\langle\psi| - |\psi^\perp\rangle\langle\psi^\perp|\right)$

Measurement with observable $M_{\psi,\lambda_\psi,\lambda_{\psi^\perp}}$

- State before measurement: $|\chi\rangle = \cos\frac{\theta}{2}|\psi\rangle + e^{i\varphi}\sin\frac{\theta}{2}|\psi^\perp\rangle$
- Measured values and final states:
  - λ_ψ with probability $p_{\chi\to\psi} = \langle\chi|\psi\rangle\langle\psi|\chi\rangle = \cos^2\frac{\theta}{2}$ and final state $|\psi\rangle$
    - With $|\chi\to\psi\rangle = |\psi\rangle\langle\psi||\chi\rangle = \langle\psi|\chi\rangle|\psi\rangle$, $p_{\chi\to\psi} = \langle\chi\to\psi|\chi\to\psi\rangle$ and final state $\frac{1}{\sqrt{p_{\chi\to\psi}}}|\chi\to\psi\rangle$
  - $\lambda_{\psi^\perp}$ with probability $p_{\chi\to\psi^\perp} = \langle\chi|\psi^\perp\rangle\langle\psi^\perp|\chi\rangle = \sin^2\frac{\theta}{2}$ and final state $|\psi^\perp\rangle$
    - With $|\chi\to\psi^\perp\rangle = |\psi^\perp\rangle\langle\psi^\perp||\chi\rangle = \langle\psi^\perp|\chi\rangle|\psi^\perp\rangle$, $p_{\chi\to\psi} = \langle\chi\to\psi^\perp|\chi\to\psi^\perp\rangle$ and final state $\frac{1}{\sqrt{p_{\chi\to\psi^\perp}}}|\chi\to\psi^\perp\rangle$

Quantum states are relative to observers!

Pairs of qubits and entanglement: the 4 Bell states

Pair of qubits:

- $\mathbf{Vect}(\mathbb{C}^2 \times \mathbb{C}^2) = \mathbb{C}^2 \otimes \mathbb{C}^2 = \mathbb{C}^4$
- Computational basis: $\{|0\rangle, |1\rangle\} \times \{|0\rangle, |1\rangle\} = \{|0\rangle \otimes |0\rangle, |0\rangle \otimes |1\rangle, |1\rangle \otimes |0\rangle, |1\rangle \otimes |1\rangle\}$
- Some other bases: $\{|\varphi\rangle, |\varphi^\perp\rangle\} \times \{|\psi\rangle, |\psi^\perp\rangle\} = \{|\varphi\rangle \otimes |\psi\rangle, |\varphi\rangle \otimes |\psi^\perp\rangle, |\varphi^\perp\rangle \otimes |\psi\rangle, |\varphi^\perp\rangle \otimes |\psi^\perp\rangle\}$
- Another basis: the 4 Bell states $BSB = \{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}$
  - $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$
  - $|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle)$
  - $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle)$
  - $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle)$
  - One can prove that $\forall\Phi \in BSB, \forall\varphi \in \mathbb{C}^2, \forall\psi \in \mathbb{C}^2, |\Phi\rangle \neq |\varphi\rangle \otimes |\psi\rangle$
  - $\Phi \in BSB$ is a (maximally) entangled state for a pair of qubits

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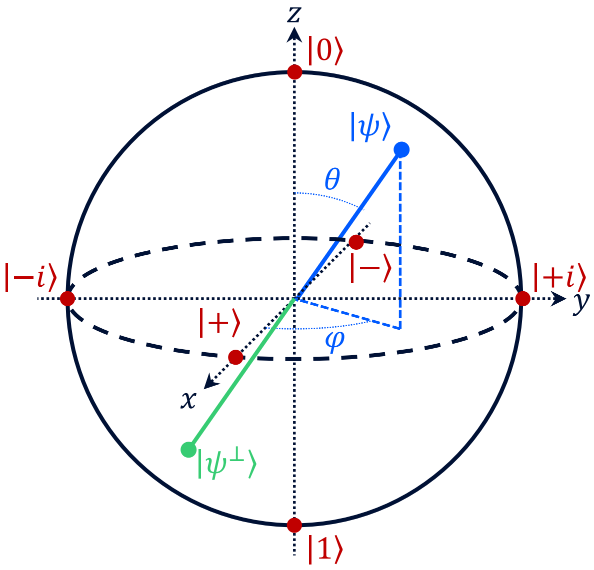
Entanglement: $|\Psi^-\rangle$ Bell state symmetry



"Central symmetry" of the bell state $|\Psi^-\rangle$

- Bell state $|\Psi^-\rangle$: $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle)$
- Computational basis $(|0\rangle, |1\rangle)$ in the basis $(|\psi\rangle, |\psi^\perp\rangle)$:

$$|0\rangle = \cos\frac{\theta}{2}|\psi\rangle - e^{-i\beta}e^{i\varphi}\sin\frac{\theta}{2}|\psi^\perp\rangle \qquad \textbf{(1')}$$

$$|1\rangle = e^{-i\varphi}\sin\frac{\theta}{2}\,|\psi\rangle + e^{-i\beta}\cos\frac{\theta}{2}|\psi^\perp\rangle \qquad \textbf{(2')}$$

- Computational basis $(|0\rangle, |1\rangle)$ in the basis $(|\psi\rangle, |\psi^\perp\rangle)$:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(\cos\frac{\theta}{2}|\psi\rangle - e^{-i\beta}e^{i\varphi}\sin\frac{\theta}{2}|\psi^\perp\rangle\right) \otimes \left(e^{-i\varphi}\sin\frac{\theta}{2}\,|\psi\rangle + e^{-i\beta}\cos\frac{\theta}{2}|\psi^\perp\rangle\right)$$

$$-\frac{1}{\sqrt{2}}\left(e^{-i\varphi}\sin\frac{\theta}{2}\,|\psi\rangle + e^{-i\beta}\cos\frac{\theta}{2}|\psi^\perp\rangle\right) \otimes \left(\cos\frac{\theta}{2}|\psi\rangle - e^{-i\beta}e^{i\varphi}\sin\frac{\theta}{2}|\psi^\perp\rangle\right)$$

$$= 0 \cdot |\psi\rangle \otimes |\psi\rangle + \frac{e^{-i\beta}\left(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right)}{\sqrt{2}}|\psi\rangle \otimes |\psi^\perp\rangle - \frac{e^{-i\beta}\left(\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}\right)}{\sqrt{2}}|\psi^\perp\rangle \otimes |\psi\rangle + 0 \cdot |\psi^\perp\rangle \otimes |\psi^\perp\rangle$$

- Choice $e^{-i\beta} = 1 \Rightarrow \forall\psi \in \mathrm{P}(\mathbb{C}^2), |\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle \otimes |\psi^\perp\rangle - |\psi^\perp\rangle \otimes |\psi\rangle)$

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Entanglement: $|\Psi^-\rangle$ Bell state measurements

State before measurement: $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\varphi\rangle \otimes |\varphi^\perp\rangle - |\varphi^\perp\rangle \otimes |\varphi\rangle) = \frac{1}{\sqrt{2}}(|\psi\rangle \otimes |\psi^\perp\rangle - |\psi^\perp\rangle \otimes |\psi\rangle)$

Measurement of the 1st qubit in state $\varphi \in \mathrm{P}(\mathbb{C}^2)$:

- $|\Psi^- \to \varphi \otimes ?\rangle = (|\varphi\rangle\langle\varphi| \otimes Id_2)|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\varphi\rangle\langle\varphi||\varphi\rangle \otimes |\varphi^\perp\rangle - |\varphi\rangle\langle\varphi||\varphi^\perp\rangle \otimes |\varphi\rangle) = \frac{1}{\sqrt{2}}|\varphi\rangle \otimes |\varphi^\perp\rangle$
- The measured state for the 1st qubit is $|\varphi\rangle$ with probability $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$
- The state of the pair after measurement is $|\varphi\rangle \otimes |\varphi^\perp\rangle$ (entanglement has disappeared…)
- The inferred state for the 2nd qubit is $|\varphi^\perp\rangle$

Measurement of the 2nd qubit in state $\psi^\perp \in \mathrm{P}(\mathbb{C}^2)$:

- $|\Psi^- \to ? \otimes \psi^\perp\rangle = (Id_2 \otimes |\psi^\perp\rangle\langle\psi^\perp|)|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle \otimes |\psi^\perp\rangle\langle\psi^\perp||\psi^\perp\rangle - |\psi^\perp\rangle \otimes |\psi^\perp\rangle\langle\psi^\perp||\psi\rangle) = \frac{1}{\sqrt{2}}|\psi\rangle \otimes |\psi^\perp\rangle$
- The measured state for the 2nd qubit is $|\psi^\perp\rangle$ with probability $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$
- The state of the pair after measurement is $|\psi\rangle \otimes |\psi^\perp\rangle$ (entanglement has disappeared…)
- The inferred state for the 1st qubit is $|\psi\rangle$

Quantum states are relative to observers!

EPR experiment: description and paradox

EPR experiment

- Pair of qubits in state $|\Psi^{-}\rangle$
- Alice (observer A) and Bob (observer B) are in different locations
  - Distance $d_{A\leftrightarrow B}$ such as $\Delta t_{A\leftrightarrow B} = \frac{d_{A\leftrightarrow B}}{c}$ is not negligible
- Alice measure the 1st qubit in state $\varphi \in \mathrm{P}(\mathbb{C}^2)$ at time t_A in her inertial frame of reference R
  - Time t'_A in Bob's inertial reference frame R'
  - She received (at the speed of light) Bob's measurement outcome at time $t_{B\to A} > t_A$
  - Thus, the projected state after her measurement is $|\Psi^- \to \varphi \otimes ?\rangle_{Alice} = \frac{1}{\sqrt{2}}|\varphi\rangle \otimes |\varphi^{\perp}\rangle$
- Bob measure the 2nd qubit in state $\psi^{\perp} \in \mathrm{P}(\mathbb{C}^2) \setminus \{\varphi, \varphi^{\perp}\}$ at time t'_B in its inertial frame of reference R'
  - Time t_B in Alice's inertial reference frame R'
  - He received (at the speed of light) Alice's measurement outcome at time $t'_{A\to B} > t'_B$
  - Thus, the projected state after his measurement is $|\Psi^- \to ? \otimes \psi^{\perp}\rangle_{Bob} = \frac{1}{\sqrt{2}}|\psi\rangle \otimes |\psi^{\perp}\rangle$
- Paradox = The states after their respective measurements are different:
  $|\varphi\rangle \otimes |\varphi^{\perp}\rangle \neq |\psi\rangle \otimes |\psi^{\perp}\rangle \Rightarrow$ Who is right, Alice or Bob?

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EPR experiment: observer-dependent time ⇒ observer-dependent state

Usual interpretation: Alice (or Bob) measures first and instantaneous change of remote qubit

- Special Relativity ⇒ time depends on the observers' inertial frames of reference R and R'
- We can choose R (resp. R') such as $t_B > t_A$, $t_B < t_A$ or even $t_B = t_A$ (resp. $t'_B > t'_A$, $t'_B < t'_A$ or $t'_B = t'_A$)
- This implies the observer-dependence of the state!

Quantum physics formalism does not care about the order of separated measurements:

$$\left(Id_2 \otimes |\psi^\perp\rangle\langle\psi^\perp|\right) \times (|\varphi\rangle\langle\varphi| \otimes Id_2) = (|\varphi\rangle\langle\varphi| \otimes Id_2) \times \left(Id_2 \otimes |\psi^\perp\rangle\langle\psi^\perp|\right) = \left(|\varphi\rangle\langle\varphi| \otimes |\psi^\perp\rangle\langle\psi^\perp|\right)$$

- The projected state of the pair of qubits for Alice after $t_{B\to A}$ is:

$$|\Psi^- \to \varphi \otimes \psi^\perp\rangle_{Alice} = \left(Id_2 \otimes |\psi^\perp\rangle\langle\psi^\perp|\right)\left(\frac{1}{\sqrt{2}}|\varphi\rangle \otimes |\varphi^\perp\rangle\right) = \frac{\langle\psi^\perp|\varphi^\perp\rangle}{\sqrt{2}}|\varphi\rangle \otimes |\psi^\perp\rangle$$

- The projected state of the pair of qubits for Bob after $t'_{A\to B}$ is:

$$|\Psi^- \to \varphi \otimes \psi^\perp\rangle_{Bob} = (|\varphi\rangle\langle\varphi| \otimes Id_2)\left(\frac{1}{\sqrt{2}}|\psi\rangle \otimes |\psi^\perp\rangle\right) = \frac{\langle\varphi|\psi\rangle}{\sqrt{2}}|\varphi\rangle \otimes |\psi^\perp\rangle$$

$$\Rightarrow |\Psi^- \to \varphi \otimes \psi^\perp\rangle_{Alice} = |\Psi^- \to \varphi \otimes \psi^\perp\rangle_{Bob} = \frac{\langle\varphi|\psi\rangle}{\sqrt{2}}|\varphi\rangle \otimes |\psi^\perp\rangle$$

because one can check that $\langle\psi^\perp|\varphi^\perp\rangle = \langle\varphi|\psi\rangle$

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EPR experiment: summary

EPR experiment + Special Relativity $\Rightarrow$ Quantum states may be relative to the observer!

- Before the measurement: same state

$$|initial\ state\rangle_{Alice} = |initial\ state\rangle_{Bob} = |\Psi^-\rangle$$

- After local measurement and before reception of other measurement outcome: different states!

$$|\Psi^- \rightarrow \varphi \otimes ?\rangle_{Alice} = \frac{1}{\sqrt{2}}|\varphi\rangle \otimes |\varphi^\perp\rangle \neq |\Psi^- \rightarrow ? \otimes \psi^\perp\rangle_{Bob} = \frac{1}{\sqrt{2}}|\psi\rangle \otimes |\psi^\perp\rangle$$

with $\psi \in \mathrm{P}(\mathbb{C}^2) \setminus \{\varphi, \varphi^\perp\}$

- After reception of other measurement outcome: same state again

$$|\Psi^- \rightarrow \varphi \otimes \psi^\perp\rangle_{Alice} = |\Psi^- \rightarrow \varphi \otimes \psi^\perp\rangle_{Bob} = \frac{\langle\varphi|\psi\rangle}{\sqrt{2}}|\varphi\rangle \otimes |\psi^\perp\rangle$$

- See Quirk circuit https://www.ludovic-noirie.fr/QC/div/MeasurementPsi-.htm

Quantum states are relative to observers!

EPR experiment: relational interpretation of quantum physics

Carlo Rovelli's relational interpretation of quantum physics

- Carlo Rovelli, *Relational Quantum Mechanics*, 1996, https://arxiv.org/abs/quant-ph/9609002
- See also https://en.wikipedia.org/wiki/Interpretations_of_quantum_mechanics#Relational_quantum_mechanics

"Quantum mechanics is a theory about the physical description of physical systems relative to other systems, and this is a complete description of the world"

- Quantum state = mathematical modeling of the state of knowledge of the observer on the observed system
- Quantum states are relative to observers (like time in special and general relativity)
- All systems are quantum systems (main difference with Copenhagen interpretation)
- Observation = Physical interaction = Entanglement (observer out) or Measurement (observer in)

View of Asher Peres (one of the inventors of quantum teleportation)

- Asher Peres, *Quantum information and general relativity*, 2004, https://arxiv.org/abs/quant-ph/0405127
- See also https://en.wikipedia.org/wiki/Asher_Peres#Views_on_the_EPR_paradox
- Quantum state = information
- When Alice measures her qubit, absolutely nothing happens at Bob's location
- Bob needs information from Alice to change his states about his qubit

Quantum states are relative to observers!

Schrödinger's cat, interactions and measurement in quantum physics

The "observed observer" (*Relational Quantum Mechanics*, Carlo Rovelli)

- Schrödinger's cat thought experiment: cat in a box in a superposition state "dead or/and alive"
  - System S observed by observer A inside the box
  - Same system S observed by observer B outside the box
- The state is relative to the observer in this configuration too:
  - Observer A: The cat (system S) is dead $|0\rangle_S$ or alive $|1\rangle_S$, not both…
  - Observer B: The cat and the observer A are entangled in superposition state $\frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_S + |1\rangle_A|1\rangle_S\right)$
- Is there really a "measurement problem" in quantum physics?
  - Observation = Acquisition of information about a system by another system (by interaction)
  - **Measurement** = Interaction between two systems (S and A), the observer being one of them (A)
  - **Entanglement** = Interaction between two systems (S and A) observed by another system (B)
- Qubit = smallest measurement apparatus to measure another qubit: see Quirk circuit https://www.ludovic-noirie.fr/QC/div/MeasurementQubit.htm

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Conclusion

Quantum states may be relative to observers

- Case 1 (EPR-like): two distant observers on the same composite quantum system with two entangled components, each observer measuring one component and each component being "significantly" distant from the other
- Case 2 (the "observed observer"): a 1st observer observing a quantum system, a second observer observing the 1st observer and the observed quantum system (measurement vs. entanglement)

When is it relevant for case 1?

- Not relevant for quantum computers: single observer, qubits in the "same location" (30 cm = 1 ns)
- But clearly relevant in quantum distributed systems such as quantum networks / quantum internet
  - Each node observes a qubit entangled in Bell state with another qubit of another node
  - The nodes are "significantly" distant (photon transmission in fiber: 50 km = 0.25 ms)
- This effect is important for QKD application in quantum internet: statistical detection of eavesdropper
  - See https://www.ludovic-noirie.fr/QC/div/QKD.htm

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Bloch spheres